Theorem: For any integer n, one of n, n+2 or n+4 must be divisible by 3.

Proof: Choose an arbitrary integer n. 3 either divides or does not divide n. If $3 \mid n$ then the chosen n is divisible by 3. If $3 \nmid n$ then by the Division Theorem (*) n = 3k + r for some k not equal to zero where $0 < r < 3$. So r equals 1 or 2.

Add 2 to both sides of (*): $n + 2 = 3k + (r + 2)$

If $r = 1$ in (*) then we have $n + 2 = 3(k + 1)$ after factoring out 3 which by the definition of divisibility means that $3 \mid n + 2$. If r = 2 then we have $n + 2 = 3(k + 1) + 1$: (**)

Add 2 to both sides of (**): $n + 4 = 3(k + 1) + 3$

Clearly $n + 4 = 3(k + 2)$ after factoring so by the definition of divisibility $3 \mid n + 4$. This covers all cases for an arbitrarily chosen n so the result holds for all n. So for any integer n, one of n, n+2 or n+4 must be divisible by 3, proving the theorem.